ME 401
Advanced Kinematics

UNIT 4

Dynamics of mechanism:

Department of Mechanical Engineering

Force Analysis of Machinery

I. Introduction:

In a dynamic analysis, we create equations that relate force and motion of a body (as in ME 233) or in our case a mechanism or machine. These are called **equations of motion**.

$$\vec{F} = m\vec{a}$$

There are 2 directions to these problems: the **Forward Dynamics** problem, where the motion is given and the forces are to be determined, and the **Inverse Dynamics** problem, where the driving force is given and the resulting motion is to be found.

Forward Dynamics

Forward kinematics: We will call our method Kinetostatics – look for dynamic equilibrium at a specific position and time, a snapshot of the mechanism.
The equations will look like:

$$[C]\{\vec{F}\} = \{m\vec{a}\}$$

With *m*, *a* known, *F* unknown, or for a mechanism:

The eq's for force are linear and solved using linear algebra

Inverse Dynamics

Inverse dynamics: Called the time-response problem. This solves the motion of a mechanism given the input driving force. Force example, the time history of the flight of an arrow leaving a bow.

For these problems, we write equations of motion (which are now differential eqations of motion) that might look like:

$$m\ddot{\vec{x}} + c\dot{\vec{x}} + k\vec{x} = \vec{F}$$

And solve motion as a function of time through numerical integration. For ex.

$$\ddot{\vec{x}} = \frac{\left(\vec{F} - c\dot{\vec{x}} - k\dot{\vec{x}}\right)}{m},$$

$$\dot{\vec{x}} = \dot{\vec{x}}_0 + \ddot{\vec{x}}\Delta t$$

$$\vec{x} = \vec{x}_0 + \dot{\vec{x}}\Delta t$$

Summary

Forward Dynamics:

"**Kinetostatics**"

Given motion, find required driving force and all bearing reactions

Inverse Dynamics:

"**Time response**"

Given input force, solve output motion as a function of time.

Motivational slide

1. Performing force analysis of a mechanism draws on all your modeling skills:
    1. Mechanism modeling, position, velocity and acceleration analysis, force analysis
2. Goals from this section:
    1. Learn how to carry out a force analysis (kinetostatics)
    2. Apply to several examples in class and HW.
    3. Create a computer model (Matlab) for force analysis and apply to a specific problem

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Review of Dynamics

1. Newtonian mechanics:
   1. Conservation of momentum $\vec{F} = 0 \Rightarrow \vec{a} = 0$
   2. Force = rate of change of momentum $\vec{F} = \frac{d}{dt}(m\vec{v})$
   3. Action/reaction $\vec{F}_{12} = \vec{F}_{21}$
2. Corollary, Euler's Equation:
   1. Torque = rate of change of angular momentum

$$\vec{T} = \frac{d}{dt}(I\omega) = I\alpha + \omega \times I\omega$$

Review of Dynamics

1. Dynamic forces:
   1. Linear acceleration:

Acceleration of pt. P:

$$a_p = a_g + {}^{d^2}\!/_{dt^2}(r_i) = a_g + \alpha \times r_i + \omega \times \beta\omega \times r_i$$

Sum forces on particle P: $dF = a_p\, dm$

And integrate to solve:

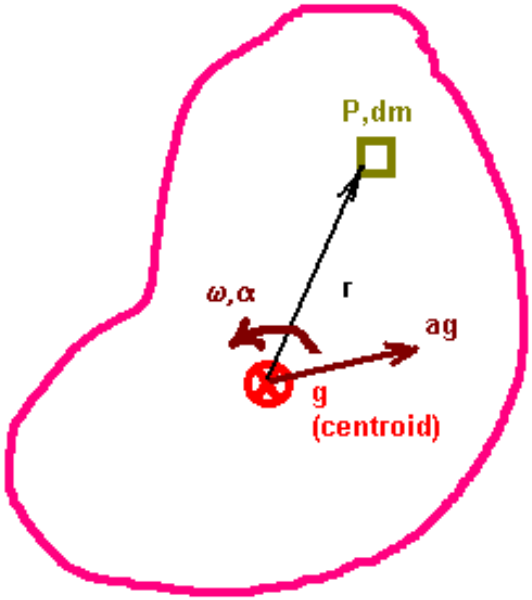
$$\sum \mathbf{F}_{ext} = \int d\mathbf{F} = \int \mathbf{a}_p\, dm = \int \left(\mathbf{a}_g + \alpha \times \mathbf{r}_i + \omega \times \omega \times \mathbf{r}_i\right) dm$$

$$\sum \mathbf{F}_{ext} = \int \mathbf{a}_g\, dm + \alpha \times \int \mathbf{r}_i\, dm + \omega \times \omega \times \int \mathbf{r}_i\, dm$$

$$\sum \mathbf{F}_{ext} = \mathbf{a}_g \int dm$$

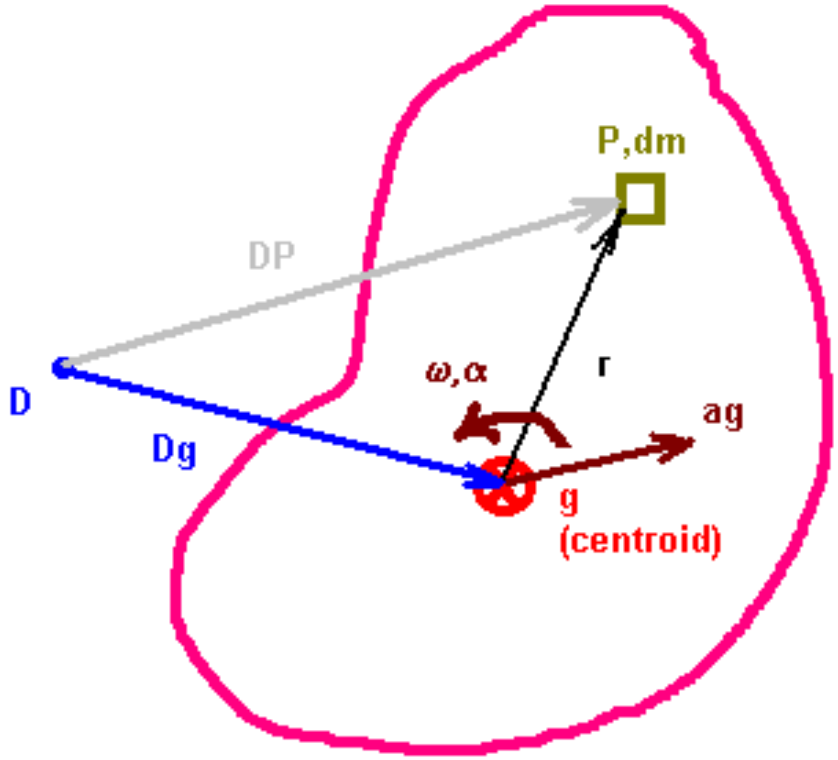
Result: **

$$\sum \mathbf{F}_{ext} = m\mathbf{a}_g$$



Review of Dynamics

2. Dynamic Moments:
    1. General layout shown in figure:



Review of Dynamics

2. Dynamic Moments (cont.):

   1. Sum moments about point D for particle P, then integrate over the body:

$$\overline{\mathbf{DP}} \times d\mathbf{F} = \overline{\mathbf{DP}} \times \mathbf{a}_\mathbf{p} dm$$

$$\mathbf{M}_{\mathbf{D,ext}} = \int \overline{\mathbf{DP}} \times d\mathbf{F} = \int \overline{\mathbf{DP}} \times \mathbf{a}_\mathbf{p} dm = \int \left(\overline{\mathbf{Dg}} + \mathbf{r}_\mathbf{i}\right) \times \left(\mathbf{a}_\mathbf{g} + \alpha \times \mathbf{r}_\mathbf{i} + \omega \times \omega \times \mathbf{r}_\mathbf{i}\right) dm$$

$$\mathbf{M}_{\mathbf{D,ext}} = \int \overline{\mathbf{DP}} \times d\mathbf{F} = \int \overline{\mathbf{DP}} \times \mathbf{a}_\mathbf{p} dm = \int \left(\overline{\mathbf{Dg}} + \mathbf{r}_\mathbf{i}\right) \times \left(\mathbf{a}_\mathbf{g} + \alpha \times \mathbf{r}_\mathbf{i} + \omega \times \omega \times \mathbf{r}_\mathbf{i}\right) dm$$

$$\mathbf{M}_{\mathbf{D,ext}} = \overline{\mathbf{Dg}} \times \mathbf{a}_\mathbf{g} \int dm + \overline{\mathbf{Dg}} \times \alpha \times \int \mathbf{r}_\mathbf{i} dm + \overline{\mathbf{Dg}} \times \omega \times \omega \times \int \mathbf{r}_\mathbf{i} dm$$

$$+ \int \mathbf{r}_\mathbf{i} \times \mathbf{a}_\mathbf{g} dm + \int \mathbf{r}_\mathbf{i} \times \alpha \times \mathbf{r}_\mathbf{i} dm + \int \mathbf{r}_\mathbf{i} \times \omega \times \omega \times \mathbf{r}_\mathbf{i} dm$$

$$\mathbf{M}_{\mathbf{D,ext}} = \overline{\mathbf{Dg}} \times m\mathbf{a}_\mathbf{g} + \alpha \int \mathbf{r}_i^2 dm$$

   2. Results: I is about c.g. $\mathbf{M}_{\mathbf{D,ext}} = \overline{\mathbf{Dg}} \times m\mathbf{a}_\mathbf{g} + I_g \alpha$

   3. If summed about c.g. (point g) $\mathbf{M}_{g,\mathbf{ext}} = I_g \alpha$

Other points to review

3. 2-force member:

A link is a 2FM if it satisfies 3 conditions:

- It has revolutes at each end
- No loads other than at the endpoints
- Mass is negligible compared to the load

All forces lie along direction of the link

Force Analysis Techniques

1. Superposition:
    1. Given a mechanism with known position, velocity, and acceleration conditions, derive Newtons equations for dynamic equilibrium. These equations are linear in the forces and therefore **Superposition** principles can be applied: Inertial and applied forces on each link can be considered individually and then superposed to determine their combined effect.
    2. This approach is good for building intuition and solving by hand.
    3. This approach can be very long
2. Matrix Method
    1. All inertial and applied forces are considered at once. The dynamic equations become coupled in the unknown forces and are solved using linear algebra techniques.
    2. Note: Why are forces linear?
    3. This approach is the best for computer application, therefore our method of choice.

Force Analysis Techniques

3. Energy method (Virtual work):
    1. Here, only forces that do work on the mechanism are considered. An equation of conservation of energy is written that results in 1 scalar equation with 1 unknown (for a 1 dof system)
    2. This is the easiest method if only the input force is required.

Superposition

1. To find driving and bearing forces using superposition, break the problem into *n-1* parts. For ex.
    1. Find Tin' = due to forces on link *n*
    2. Find Tin'' = due to forces on link *n-1*

    3. Find Tin'''''' = due to forces on link 2
2. Notes:
    1. Notation: F14 = force of link 1 on link 4
    2. Be careful with signs
    3. F14=-F41,
    4. But, remember to either switch the sign, or the direction of the vector, but not both
    5. **T=r**x**F**

Superposition: Procedure

1. Break the problem into *n-1* parts, *n* = number of links.
2. Start with part 1
    1. Draw FBD's of the extreme link
    2. Include all forces **for Part 1 ONLY**
    3. Look for 2FM's to reduce unknowns
    4. Solve unknown reaction forces as 3 eq's, 3 uk's (in general)
    5. 3 equations are:

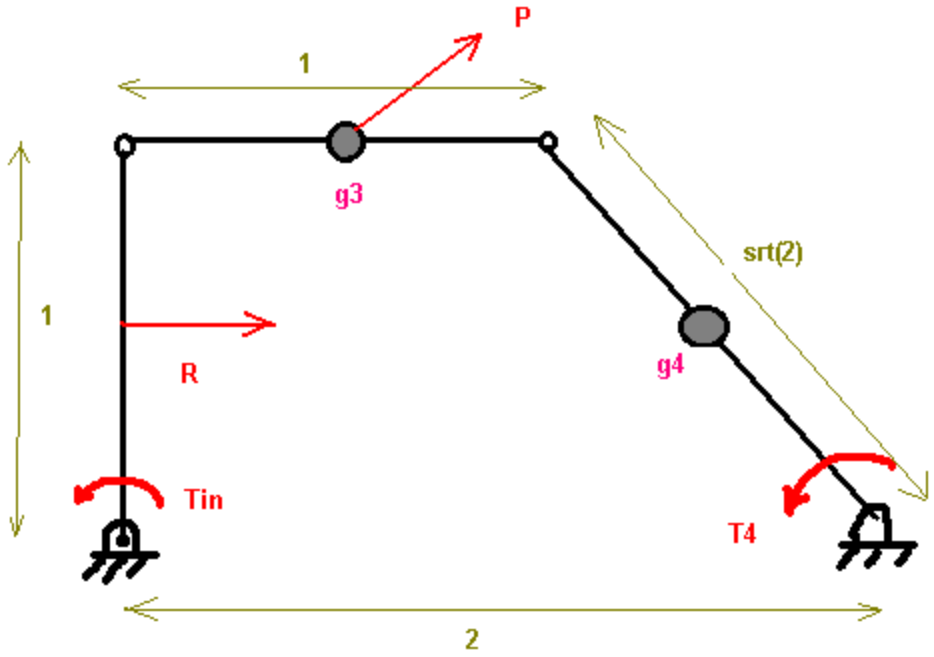
$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$
$$\sum M_g = I_g \alpha$$

Superposition: Procedure (cont.)

6. Move to the next link, transferring forces from the previous as,
    1. F23=-F32 (or, changing the directions on the vectors and keeping the magnitude)
7. Continue to the driving link, solve for Tin’

2. Move to part II
    1. Repeat, solving Tin’’
3. Continue for all parts, 1 to *n*-1
4. Find the total reactions as:
    1. Tin(tot) = Tin’+Tin’’+Tin’’’+ …
    2. F12x(tot)= F12x’+ F12x’’+…
    3. Etc.
5. Note, consistent directions for all the reaction forces must be maintained in all parts.

Superposition: Example

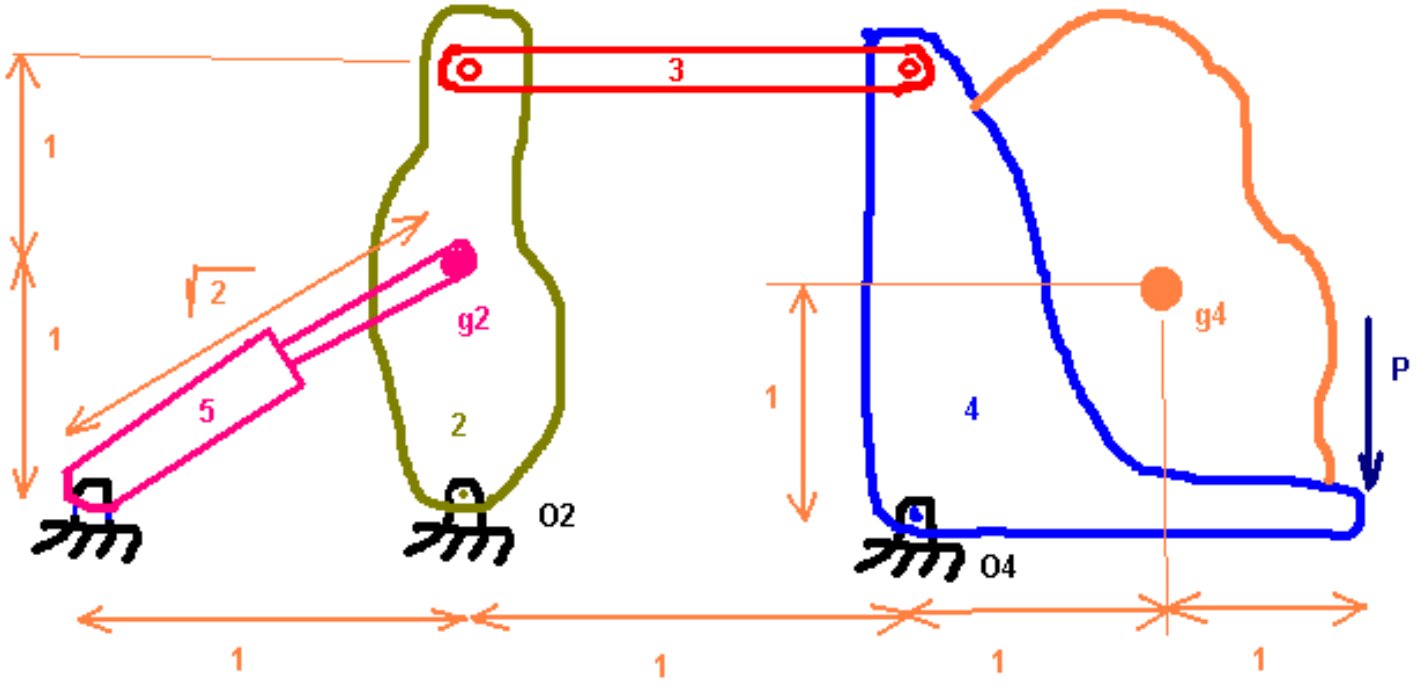


Given:

- P=10,45deg, R=10i, T4=5k
- m2=0,m3=1,m4=sqrt2
- I2=0, I3=1, I4=1
- ag3=-1i+1j, ag4=-1j
- $\alpha 3=0$, $\alpha 4=1$

Find: Tin and reactions at grd. bearings

Superposition: Example 2



- Hydraulic-powered scoop with a load in bucket (link 4)

Superposition: Example 2

- Links 2 & 4 have mass
  - m2 = 10 kg, I2 = 1 kgm^2
  - m4 = 100 kg I4 = 10 kgm^2
- Consider the cylinder, 5 as a 2 FM. Given the following motion information, find the input cylinder force and the bearing reactions at points O2 and O5.
- Motion info:
  - ω2=1, α2=1 rad/s^2 ω4=1, α4=1 rad/s^2
  - Vg2=-1i+0j m/s Vg4=-1i+1j m/s
  - Ag2=01i-1j m/s^2 Ag4=-2i+0j m/s^2
  - Vp=2j
- Force on bucket:
  - P = 100N

Force Analysis using the Matrix Method

In the matrix method, equations of dynamic equilibrium are written for FBD's of all the links in the mechanism w/ all internal and external forces included. This results in a coupling of the unknown forces. However, the equations are linear in these forces and may be solved using linear algebra techniques.

For example:

$$[C]\begin{Bmatrix}\vec{F}\\ \vec{T}\end{Bmatrix} = \begin{Bmatrix}m\vec{a}\\ I\vec{\alpha}\end{Bmatrix}$$

Known motion info

Known coefficients

Unknown forces and torques

Solving forces:

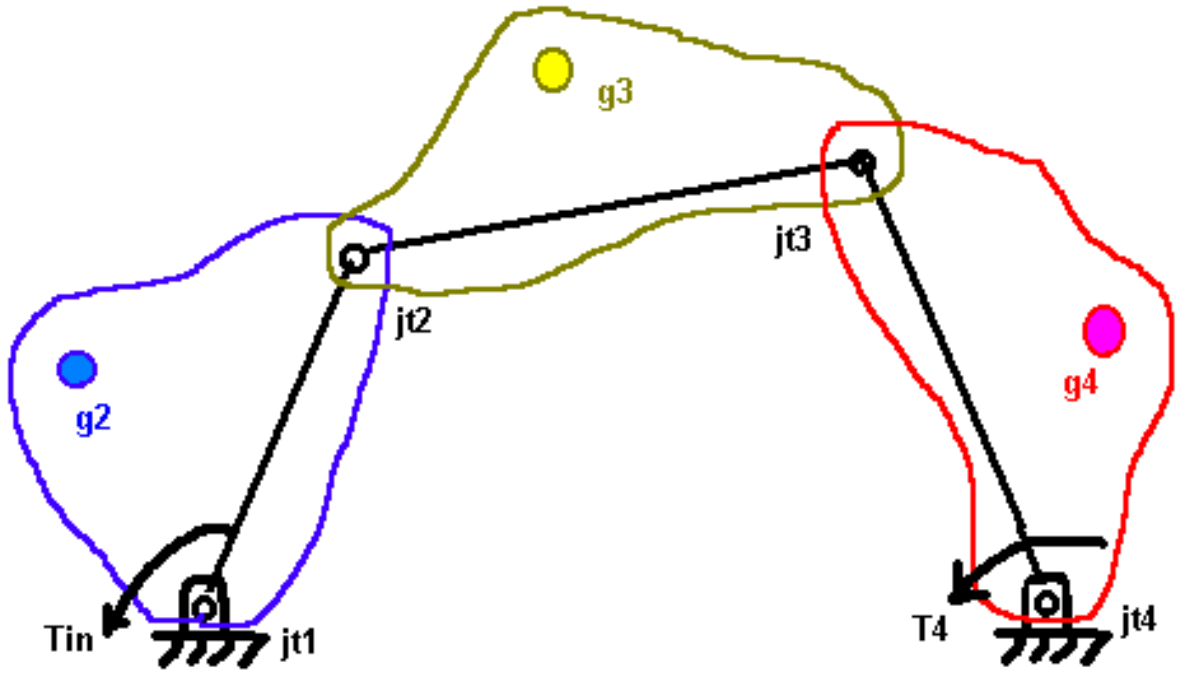
$$\begin{Bmatrix}\vec{F}\\ \vec{T}\end{Bmatrix} = [C]^{-1}\begin{Bmatrix}m\vec{a}\\ I\vec{\alpha}\end{Bmatrix}$$

Matrix Method (cont.)

In a general mechanism, there may be anywhere from 10 to 30 unknown forces to solve. ➔**Solve in Matlab** (or other computer program)

This method will be demonstrated first on a four-bar linkage.

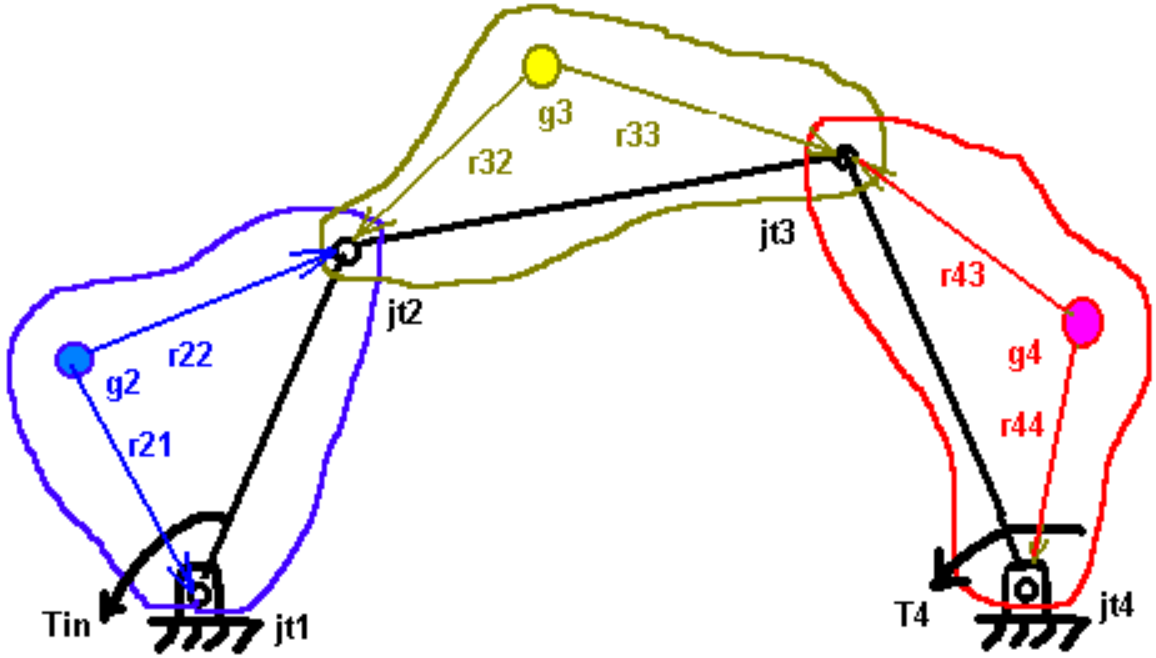
Matrix Method: Example 1



The figure above shows a general 4-bar linkage. The center of mass of each link is shown, as well as the input torque on link 2, and an applied torque (T4) on link 4.

Matrix Method: Example 1 (cont.)

Here, we have included some additional vectors to help define our problem. This leads to the following notation:



gi = center of mass of link i

jti = joint i

rij = vector from cm of i to jt. j

Fij = vector force of i on j

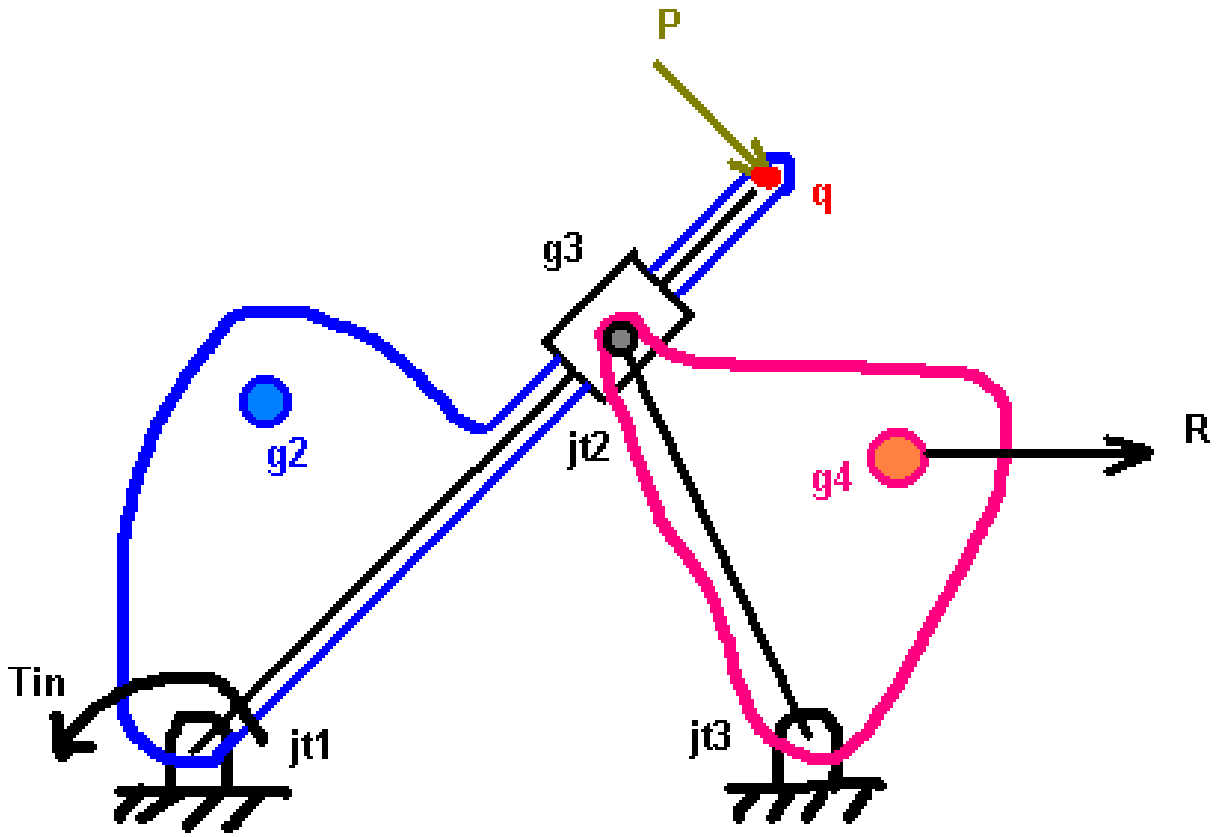
Matrix Method: Example 1 (cont.)

1. Count the number of unknowns:
   - There are two unknown forces at every 1 dof joint
   - There is one unknown force for every input
2. Count the number of equations:
   - There are three equations for each body

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$
$$\sum M_g = I_g \alpha$$

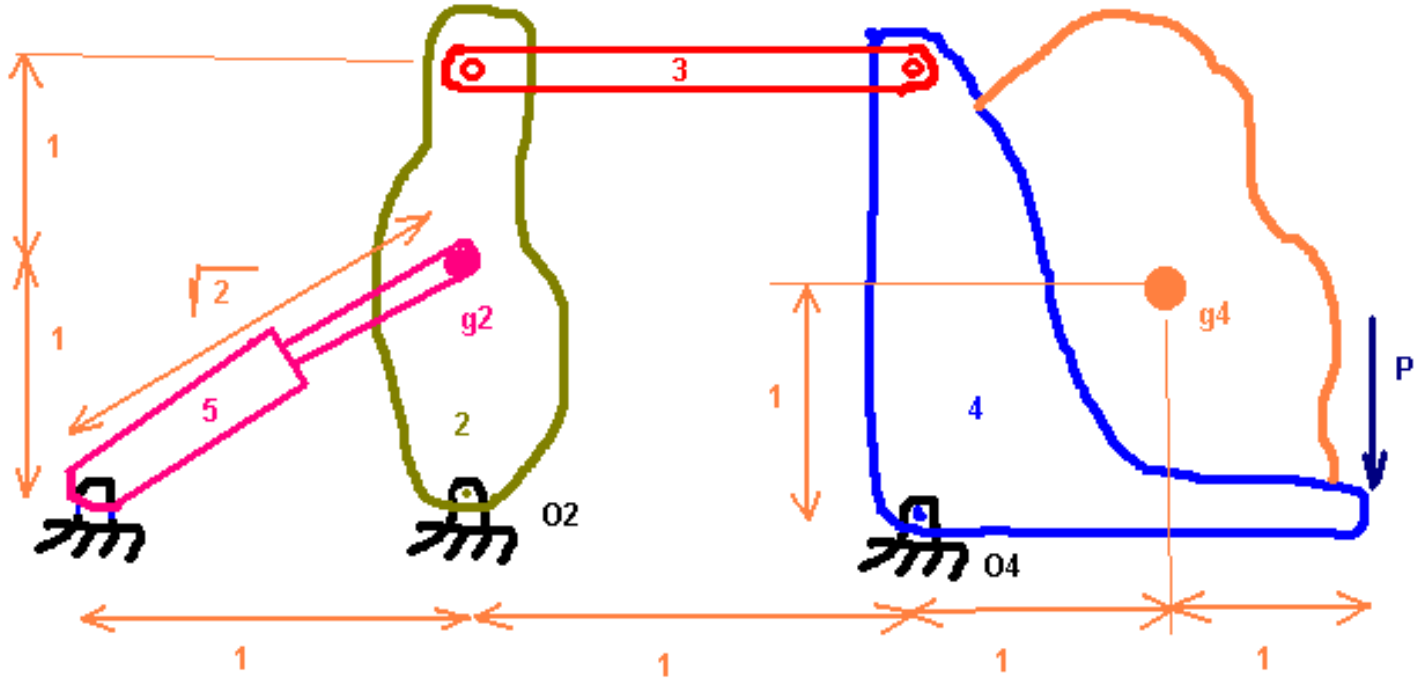
3. In this example:
   - # unknowns = 9
   - # equations = 9

Matrix Method: Example 2
Inverted Slider Crank



Solve for the input torque and all bearing reactions using the matrix method – set up the linear system of equations in matrix form.

Matrix Method: Example 3
Hydraulic-powered scoop



Matrix Method: Example 3 (cont.)
Given information:

- Links 2 & 4 have mass
  - m2 = 10 kg, I2 = 1 kgm^2
  - m4 = 100 kg I4 = 10 kgm^2
- Consider the cylinder, 5 as a 2 FM. Given the following motion information, find the input cylinder force and the bearing reactions at points O2 and O5.
- Motion info:
  - ω2=1, α2=1 rad/s^2 ω4=1, α4=1 rad/s^2
  - Vg2=-1i+0j m/s Vg4=-1i+1j m/s
  - Ag2=01i-1j m/s^2 Ag4=-2i+0j m/s^2
  - Vp=2j
- Force on bucket:
  - P = 100N

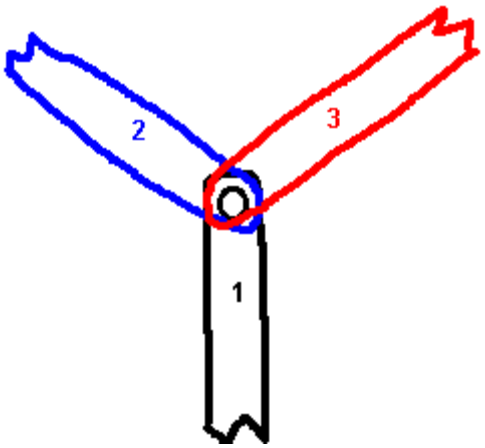
Some special cases you might see

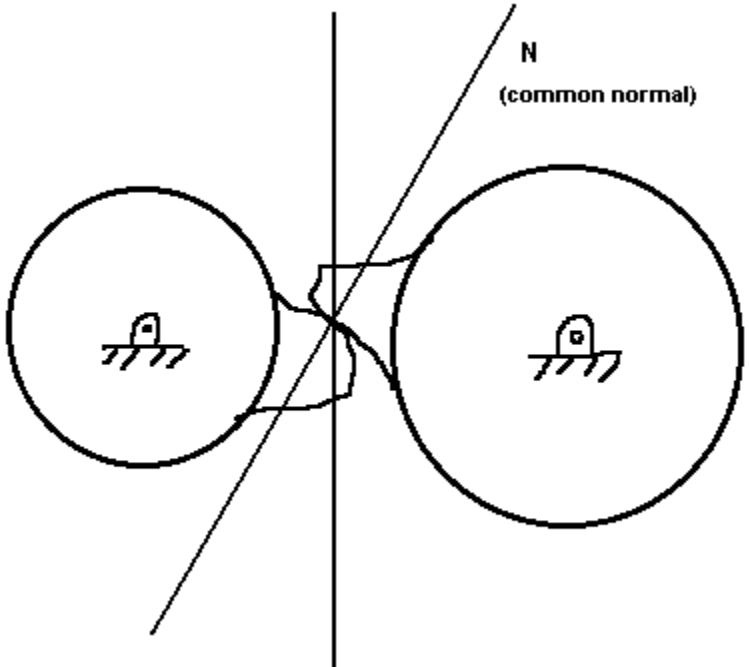
1. Multiple links @ 1 joint

At this joint, there are 2 unknown vector reactions, say F12 and F13

2. Gears:

The direction of the force between the gears is known (along the common normal), with the magnitude unknown





Special cases (cont.)

3. Hydraulic Cylinder:
    1. Model as 2 rigid bodies, to result in an a fluid pressure force, a normal-wall force, and a wall torque
    2. Model as a two force member, then there is one unknown, the force in the member (* preferred)

Force Analysis using the method of Virtual Work

1. If a rigid body is in equilibrium under the action of external forces, the total work done by these forces is zero for a small displacement of the body.
2. Work: $W = \int \mathbf{F} \cdot d\mathbf{x}, \quad W = \int \mathbf{T} \cdot d\boldsymbol{\theta}$

    1. With **F**, **x**, **T**, **q**, vectors and W a scalar.
    2. To indicate that we are dealing with infinitesimal displacements (virtual displacements), use the notation:

$$\delta W = \mathbf{F} \cdot \delta\mathbf{x} \quad \delta W = \mathbf{T} \cdot \delta\boldsymbol{\theta}$$

3. Now apply the virtual work definition:

$$\delta W = \sum_i \mathbf{F}_i \cdot \delta\mathbf{x}_i + \sum_j \mathbf{T}_j \cdot \delta\boldsymbol{\theta}_j \quad = 0$$

Virtual Work (cont.)

4. If we divide the virtual work by a small time step, we get:

$$\sum_i \mathbf{F}_i \cdot \mathbf{v}_i + \sum_j \mathbf{T}_j \cdot \boldsymbol{\omega}_j = 0$$

5. These are all external torques and forces on the body, and include inertial forces and gravity. Rewrite, to clearly show this as:

$$\sum_i \mathbf{F}_i \cdot \mathbf{v}_i + \sum_j \mathbf{T}_j \cdot \boldsymbol{\omega}_j + \sum_k \mathbf{F}_{0k} \cdot \mathbf{v}_k + \sum_l \mathbf{T}_{0l} \cdot \boldsymbol{\omega}_l = 0$$

With:

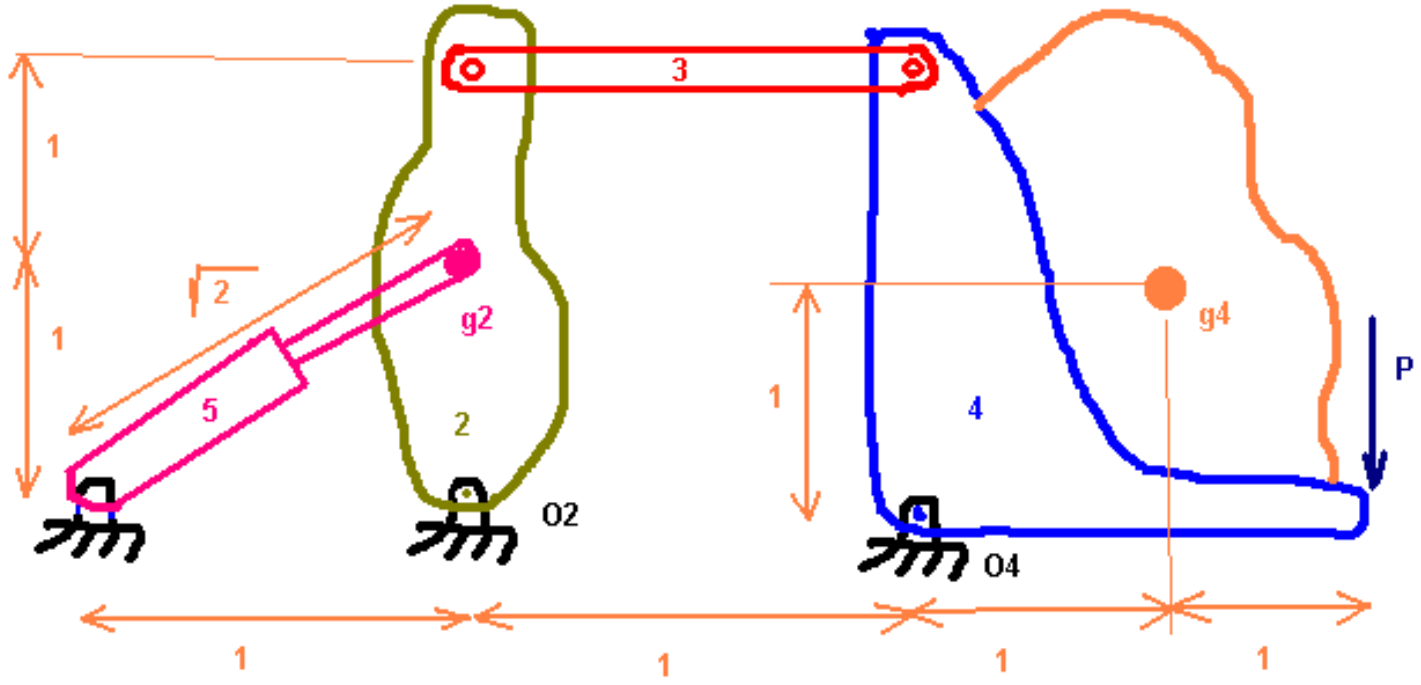
$$\mathbf{F}_{0k} = -m\mathbf{a}_{gk}, \quad \mathbf{v}_k = \mathbf{v}_{gk}$$

$$\mathbf{T}_{0l} = -I_{gl}\boldsymbol{\alpha}_l, \quad \boldsymbol{\omega}_l = \boldsymbol{\omega}_l$$

Virtual Work (cont.)

1. Notes
    1. Solve the driving force of a single dof system with a single scalar equation
    2. Internal forces (ex. Bearing reactions) cannot be solved with this technique.

Virtual Work: Example 1
Hydraulic-powered scoop



Virtual Work: Example 1 (cont.)
Given information:

- Links 2 & 4 have mass
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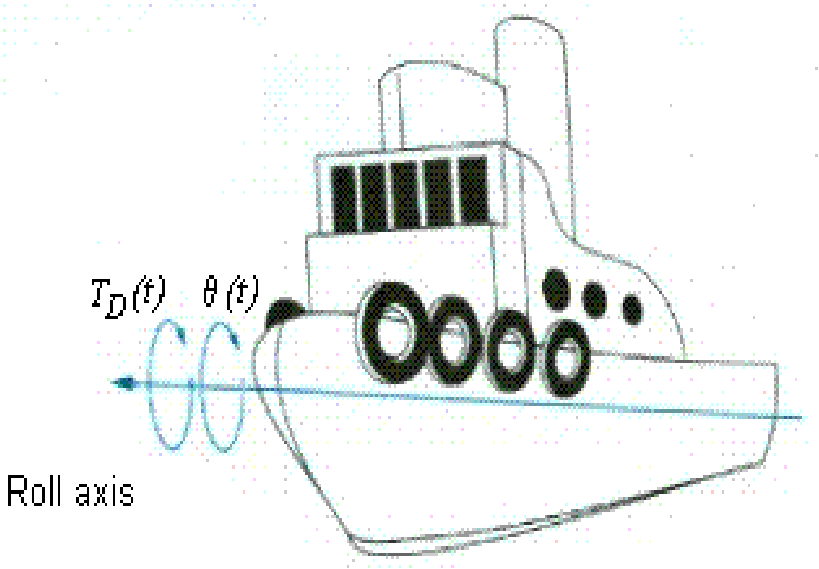
  - m4 = 100 kg I4 = 10 kgm^2
- Consider the cylinder, 5 as a 2 FM. Given the following motion information, find the input cylinder force and the bearing reactions at points O2 and O5.
- Motion info:
  - ω2=1, α2=1 rad/s^2, ω4=1, α4=1 rad/s^2
  - Vg2=-1i+0j m/s Vg4=-1i+1j m/s
  - Ag2=-1i-1j m/s^2 Ag4=-2i+0j m/s^2
  - Vp=2j, V5=1i+1j
- Force on bucket:
  - P = 100N

Problem In Ch. 4 (p. 230)

Ships in turbulent seas incur motion about their roll axis . Stabilizing fins, which can be positioned by a closed-loop control system, can be used to reduce this motion. Components describing this system include fin actuators, sensors, and roll dynamics.



Transfer Function

For roll dynamics, which relates the roll-angle output θ(s) to disturbance-torque input TD(s), is given by

$$\frac{\theta(s)}{TD(s)} = \frac{2.25}{(s2 + 0.5s + 2.25)}$$

Using MATLAB do the following

A) Find the natural frequency, damping ratio, peak time, settling time, rise time, and percent overshoot.

B) Find the analytical expression for the output response to a unit step input in voltage.

C) Plot the response found in (B).

Part A

From the denominator s2 + 0.5s +2.25 of the closed-loop transfer function T = θ(s) / TD(s), the closed-loop poles are:

-0.25 + j1.47902 and -0.25 - j1.47902.

Part A

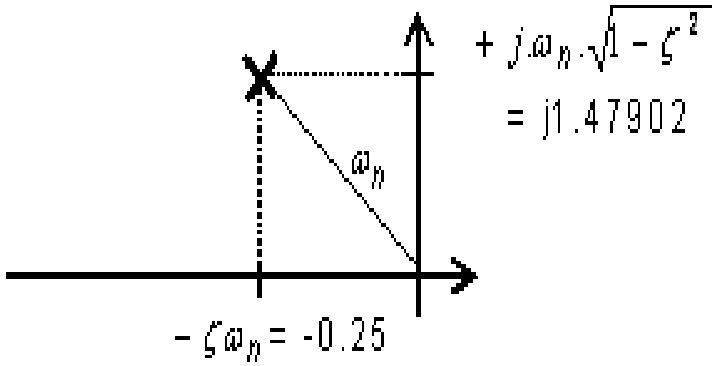
From these pole locations the two quantities describing the transient response of this second-order system are found

Natural Frequncy :

$$\omega_n = \sqrt{0.25^2 + 1.47902^2} = 1.5$$

Damping Ratio :

$$\zeta = \frac{0.25}{\omega_n} = \frac{0.25}{1.5} = 0.167$$



Part A

Percent Overshoot :

$$\%OS = e^{-(\sigma_d \pi/\omega_d)} \times 100\% = e^{-0.25\pi/1.47902} \times 100\% = 58.8\%$$

Peak Time :

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{1.47902} = 2.1241 \text{sec.}$$

Settling Time :

$$T_s \cong \frac{4}{\zeta\omega_n} = \frac{4}{0.25} = 16 \text{ sec.}$$

Rise Time :

$$T_r = \frac{1.17033}{\omega_n}$$ from look-up table (Nise 2004, p196) = 0.78022 sec.

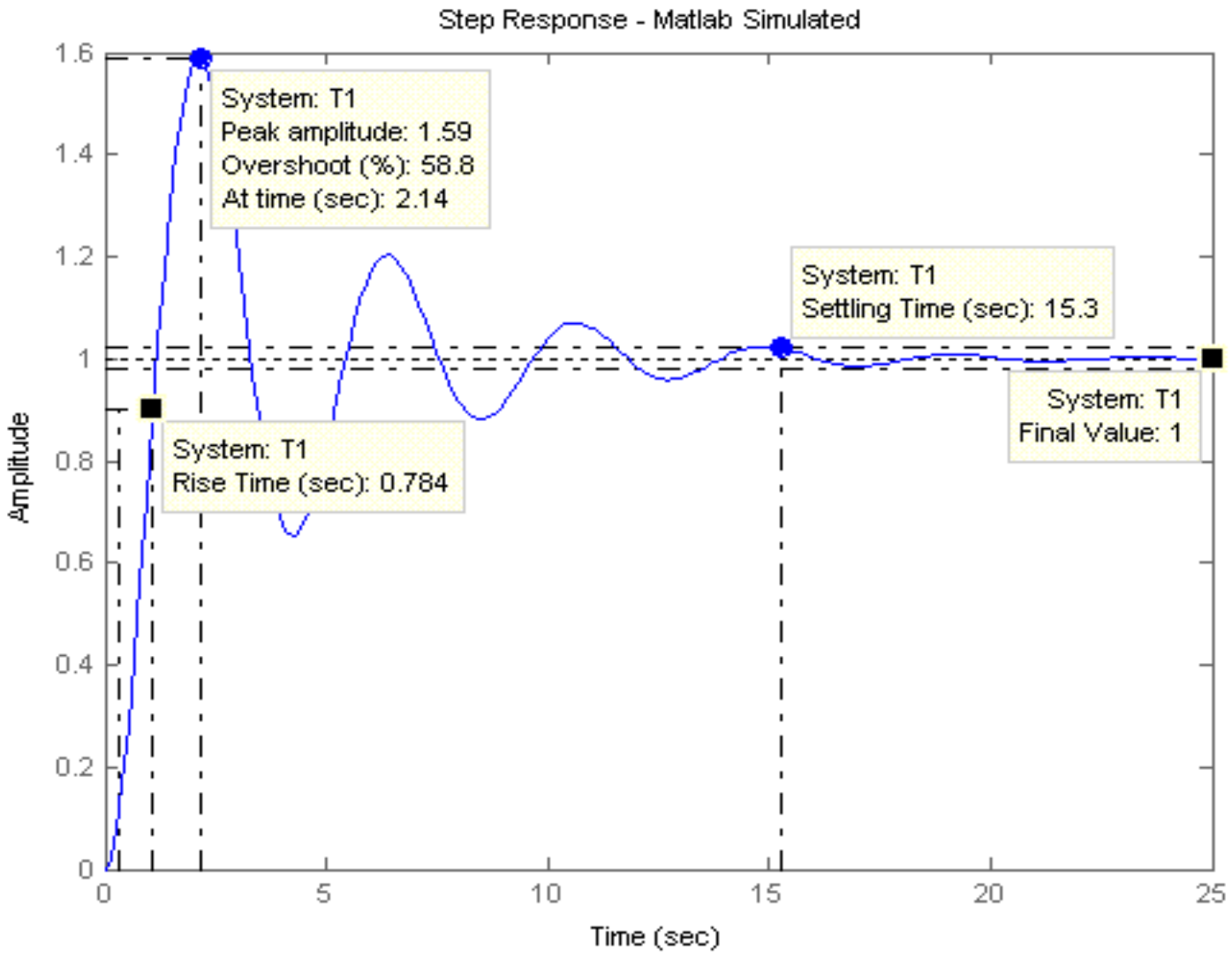
Part B

Using partial fraction expansion and unit step input, the Laplace transform of the response θ(s) is:

$$\theta(s) = \frac{2.25}{s(s^2 + 0.5s + 2.25)} = \frac{A}{s} + \frac{Bs + C}{(s^2 + 0.5s + 2.25)}$$

A = 1 , B=0.1 , C = 0.5

Part C


Step Response - Matlab Simulated
System: T1
Peak amplitude: 1.59
Overshoot (%): 58.8
At time (sec): 2.14
System: T1
Settling Time (sec): 15.3
System: T1
Final Value: 1
System: T1
Rise Time (sec): 0.784
Amplitude
Time (sec)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0
5
10
15
20
25

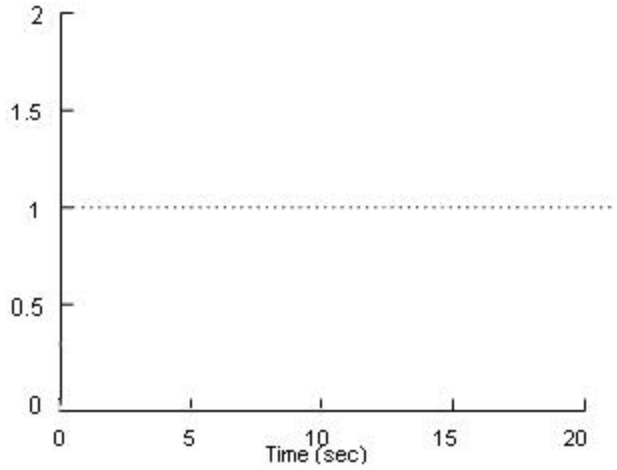
Animation

Please select an animation:

Step response for originally given system

Analytical expression for the step response

Clear graph and reset

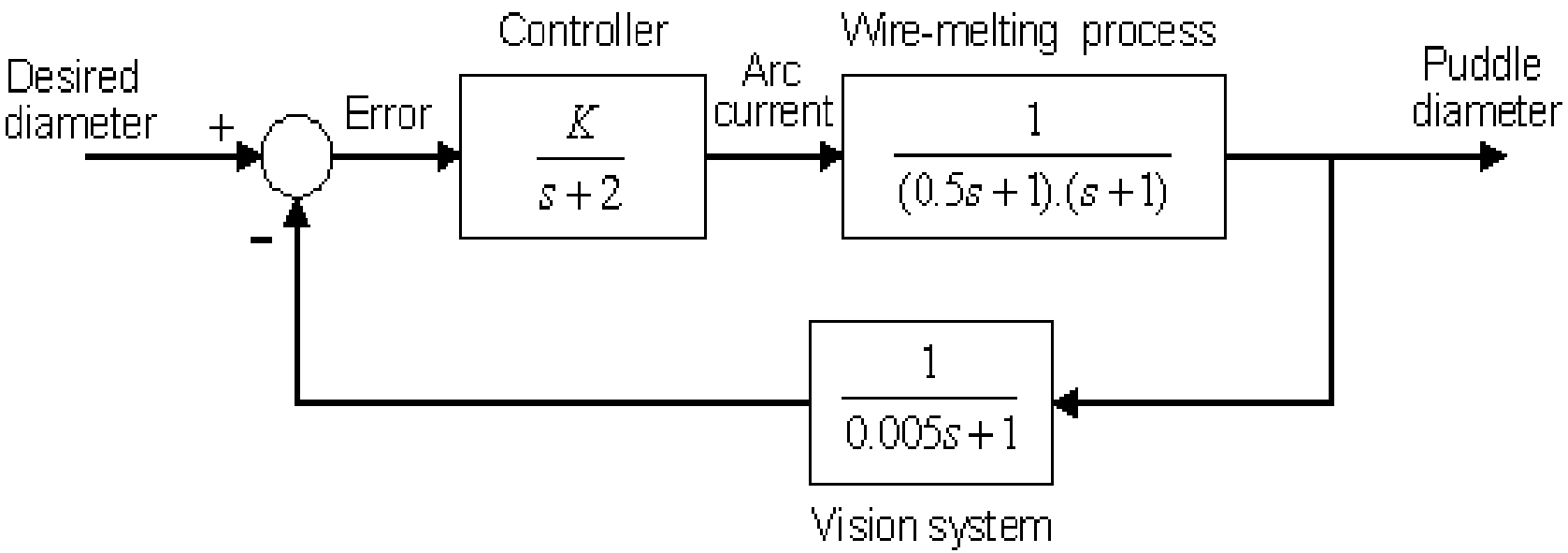


Problem 2

PROBLEM DEFINITION

Most manufacturing welding situations involve many uncertainties, including dimensions of the part, joint geometry, and the welding process itself. To ensure weld quality, sensors are therefore necessary. Some such systems, as described by figure 1, use a vision system to measure the geometry of the puddle of melted metal. Here, it is assumed that the rate of feeding the wire to be melted is constant

Block Diagram of the System



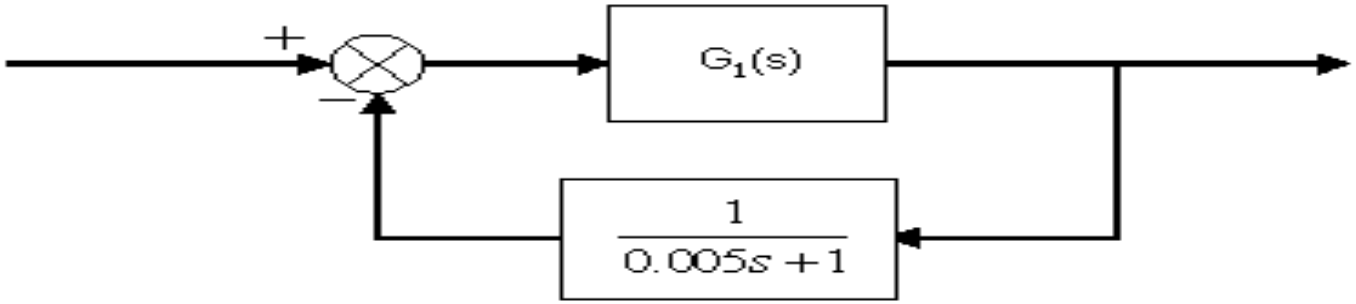
Using MATLAB do the following

A) Determine a second-order model for the closed-loop system.

B) Find the overshoot and peak time of the system with gain K = 10 using both the second-order model and original system, then compare the results. (Assume a step input.)

C) Using the second-order model, select a gain K so that the settling time is less than 4 seconds and the peak time is less than 5 seconds, while ensuring a minimal overshoot (less than 1%). Simulate the original system for this gain and compare the results

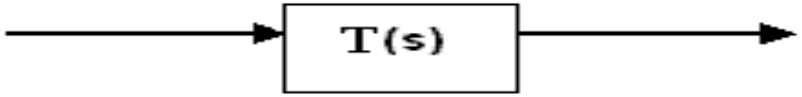
Part A)



where $G_1(s)$ = Controller × Wile-melting process

$$= \frac{1}{(s+2)} \times \frac{1}{(0.5s+1).(s+1)}$$

Finding T(s) :



$$T(s) = \frac{G_1(s)}{1 + G_1(s).H(s)} = \frac{\dfrac{K}{(s+2).(0.5s+1).(s+1)}}{1 + \dfrac{K}{(s+2).(0.5s+1).(s+1)} \times \dfrac{1}{(0.005s+1)}}$$

$$= \frac{K(0.005s+1)}{(s+2).(0.5s+1).(s+1).(0.005s+1) + K}$$

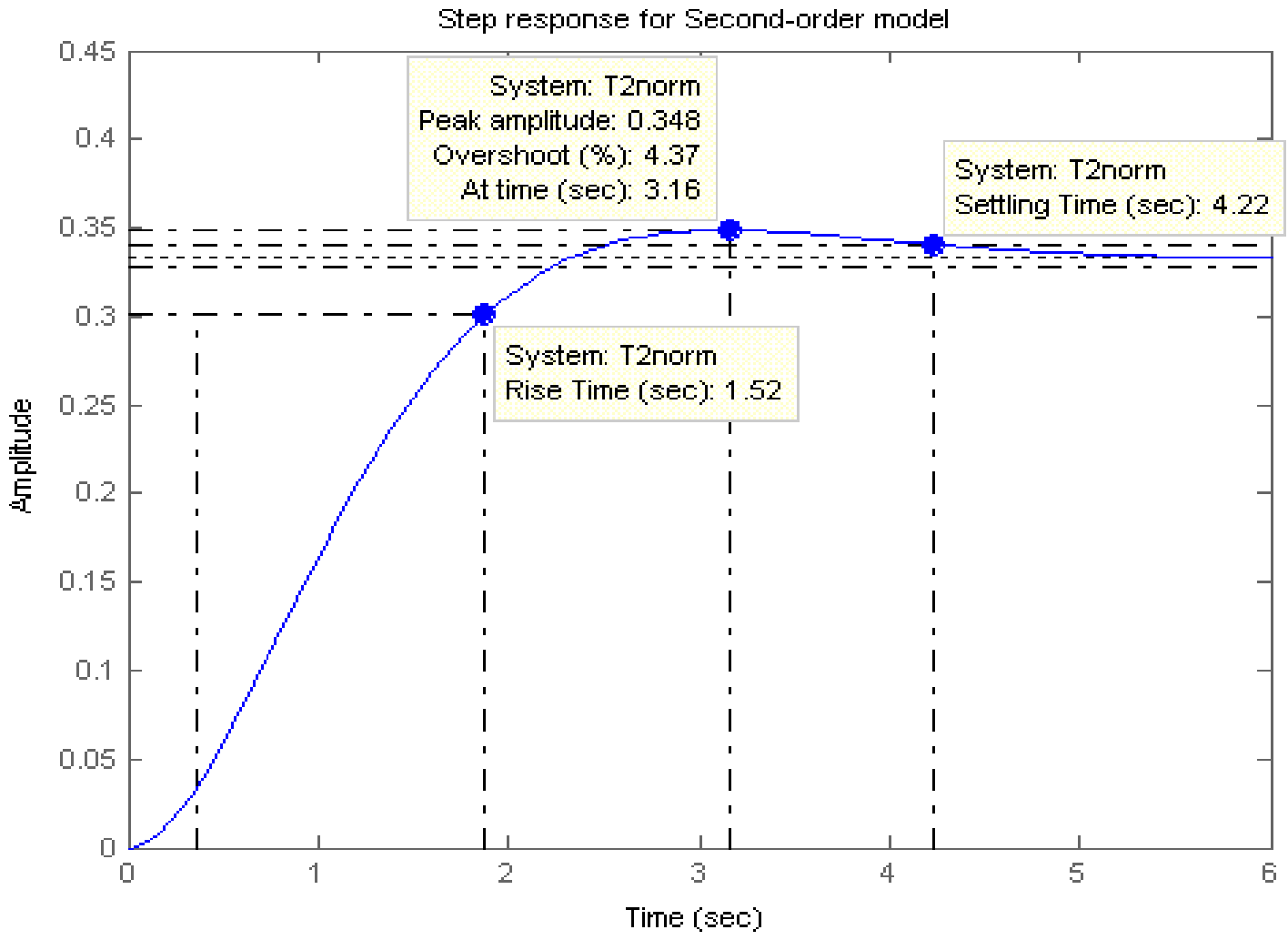
$$= \frac{0.005Ks + K}{0.0025s^4 + 0.5125s^3 + 2.52s^2 + 4.04s + 2 + K}$$

Part A)

The four poles of the closed-loop system were found to be -200, -3, -0.997+j1 and -0.997-j1. The last two poles are the dominant pair, forming the second-order model described by:

$$\frac{0.6664}{s^2 + 1.994s + 1.996}$$

Part A)


Step response for Second-order model
System: T2norm
Peak amplitude: 0.348
Overshoot (%): 4.37
At time (sec): 3.16
System: T2norm
Settling Time (sec): 4.22
System: T2norm
Rise Time (sec): 1.52
Amplitude
Time (sec)
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6

Part B)

An approximation of the overshoot and peak time for a selected value of gain K can be found from the location of the dominant poles of the closed-loop system. For poles at location $-\sigma d \pm j\omega d$, the overshoot is calculated as:

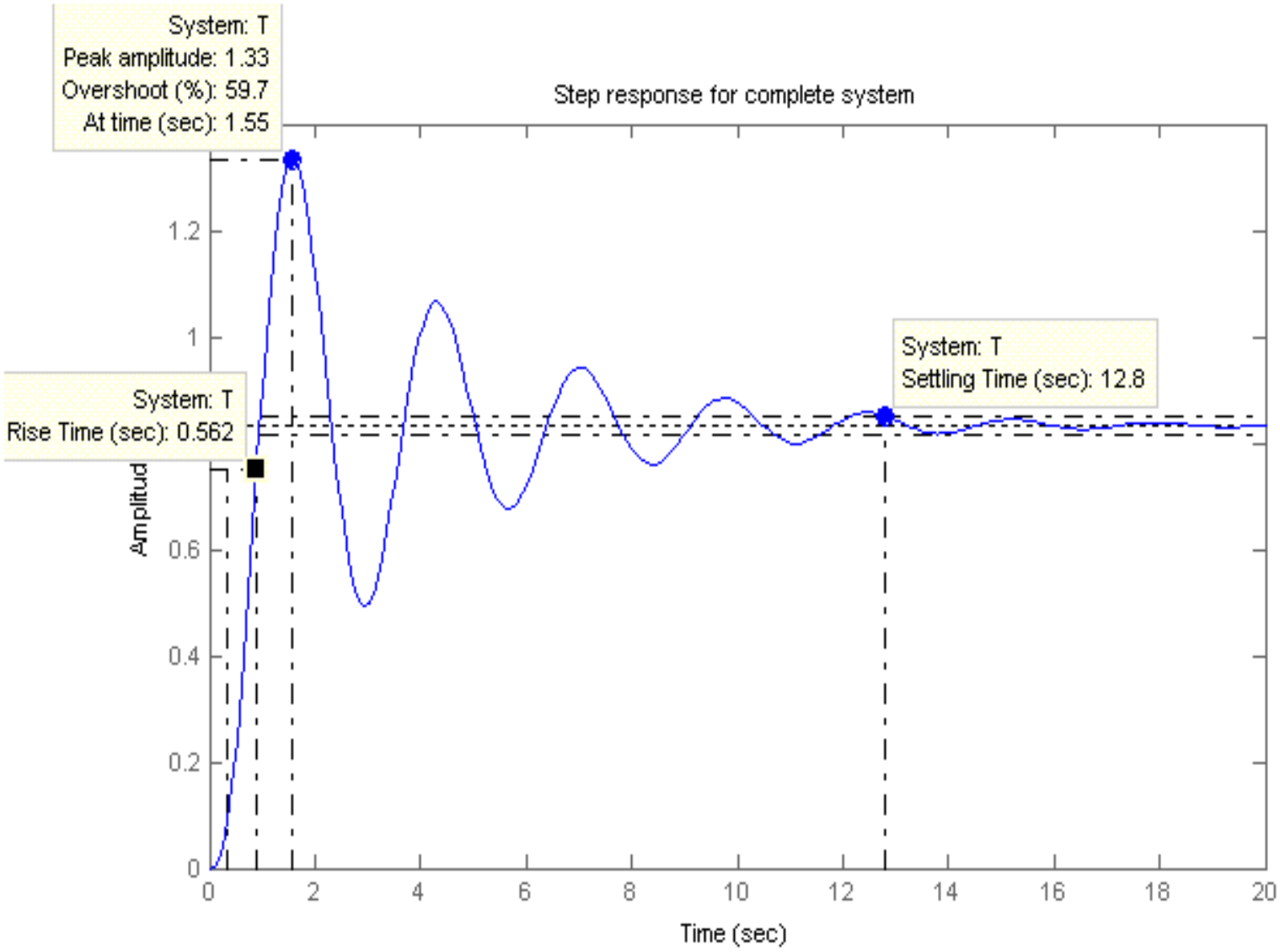
$$\%OS = \exp(-\sigma d\pi/\omega d) * 100\%$$

and peak time as:

$$Tp = \pi/\omega d.$$

Tp =1.36103466059 OS = 68.233828656

Time Response



For gain: K = 0.5

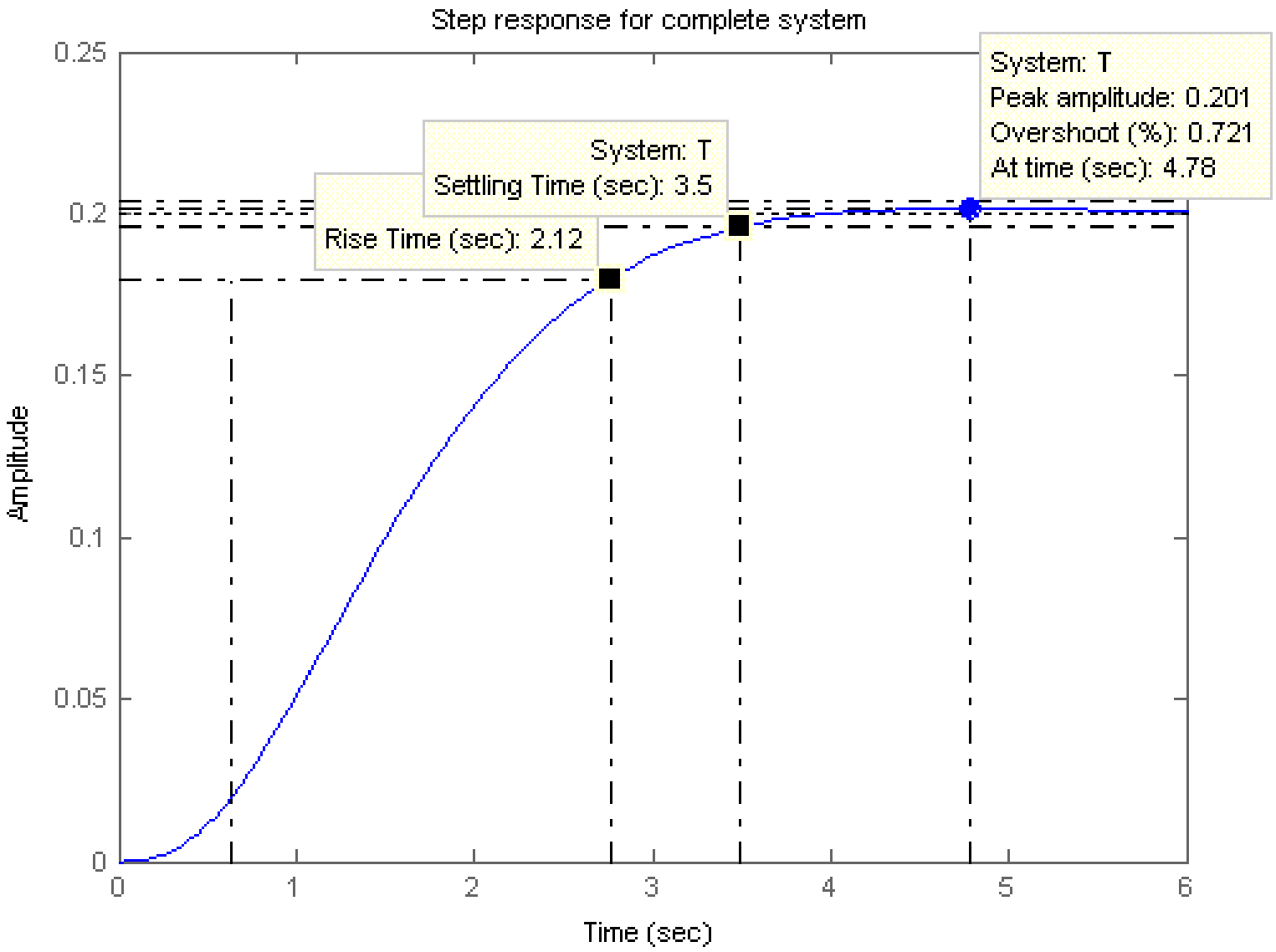
Closed-Loop System: Transfer function:

$$\frac{0.0025\,s + 0.5}{0.0025\,s^4 + 0.5125\,s^3 + 2.52\,s^2 + 4.01\,s + 2.5}$$

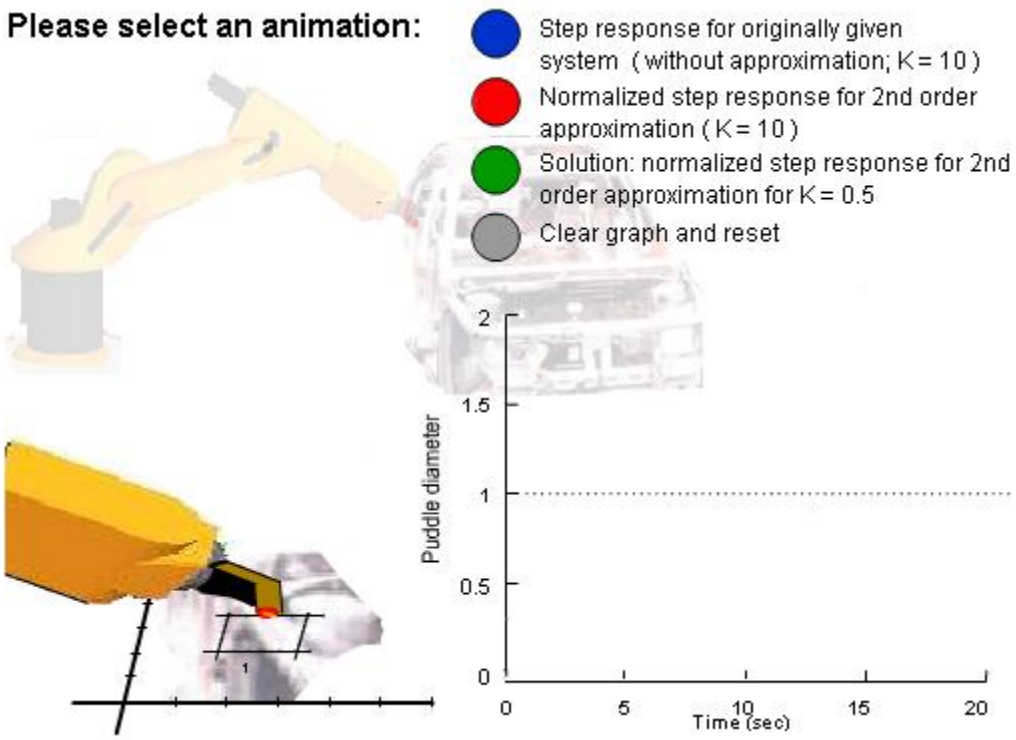
yields an estimated peak time of: Tp = 4.21018862070813

settling time of: Ts =3.5701191261793

and percent overshoot of: OS = 0.894070557339009


Step response for complete system
System: T
Peak amplitude: 0.201
Overshoot (%): 0.721
At time (sec): 4.78
System: T
Settling Time (sec): 3.5
Rise Time (sec): 2.12
Amplitude
Time (sec)
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6

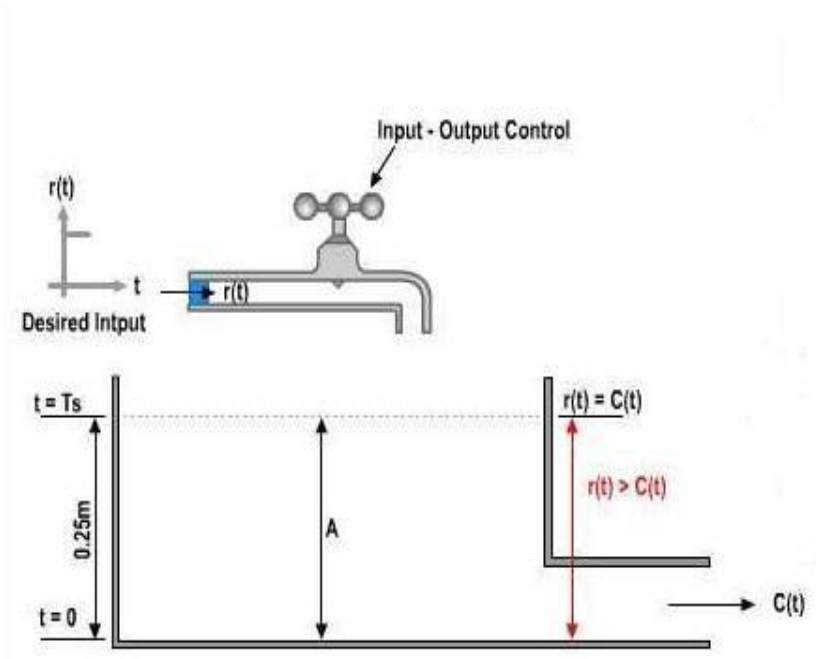
Animation


Please select an animation:
Step response for originally given system (without approximation; K = 10)
Normalized step response for 2nd order approximation (K = 10)
Solution: normalized step response for 2nd order approximation for K = 0.5
Clear graph and reset
Puddle diameter
2
1.5
1
0.5
0
0
5
10
15
20
Time (sec)

Problem 3

PROBLEM DEFINITION

$$8\frac{dc_{(t)}}{dt} + 4C_{(t)} = r_{(t)}$$



Example 1.1: Time Response

a) Find the transfer function G(s) of the system represented by the given differential equation.

b) For the obtained transfer function G(s) from part a, find the system response C(t) for unit step input r(t) = u(t) assuming zero initial conditions.

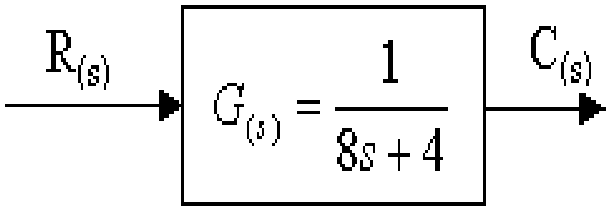
c) Find the value of gain K to yield a 2% error in the steady-state.

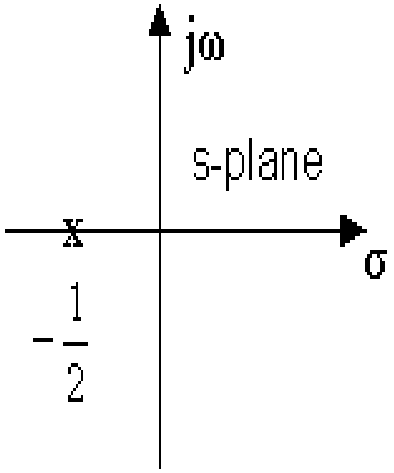
d) For the obtained gain K in part c, find the time constant t, rise time Tr and settling time Ts.

e) Using matlab plot closed-loop system step response for K = 1 and K obtained in part c.

f) Give full comment on the simulated results obtained in part e for the required values of gain K

Transfer Function





system response for unit step input

$$C(t) = \frac{1}{4} - \frac{1}{4} * e^{-\frac{1}{2}t}$$

The value of gain K to yield a 2% error in the steady-state

$$G(s)=\frac{K}{8s+4};K=1\Rightarrow e_{step}^{\infty}=\lim_{s\to 0}\frac{s*R(s)}{1+G(s)}=\frac{1}{1+\frac{1}{4}}=\frac{1}{1.25}=80\%$$

$$e_{step}^{\infty}=\lim_{s\to 0}\frac{s*R(s)}{1+G(s)}=\lim_{s\to 0}\frac{s*\frac{1}{s}}{1+\frac{K}{8s+4}}=\frac{1}{1+\frac{K}{4}}$$

$$K=\frac{4(1-0.02)}{0.02K}=\frac{3.92}{0.02}=196$$

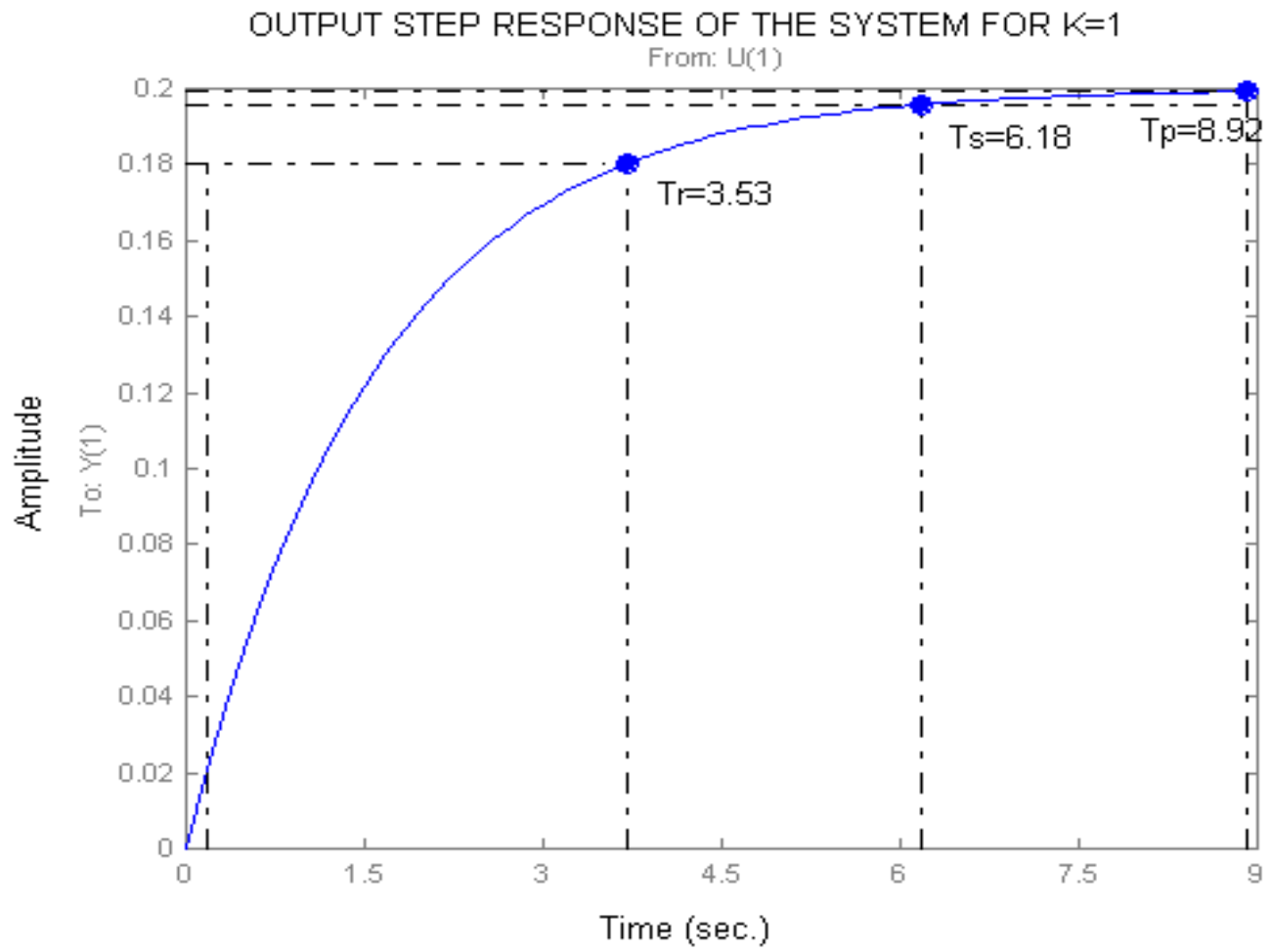
For K = 196

$$G(s) = \frac{196}{8s + 4}$$

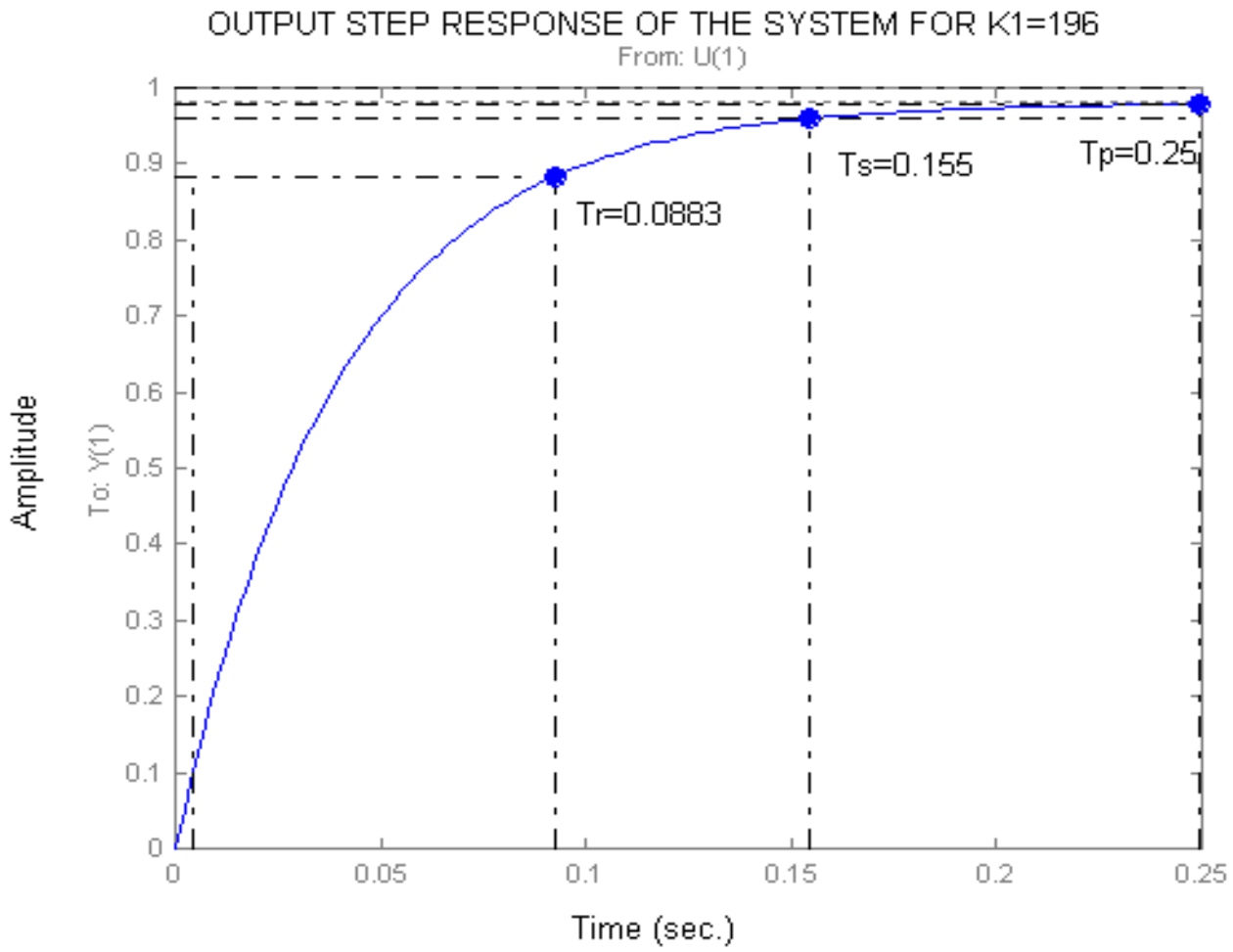
$$Tr = \frac{2.1985}{0.5} = Tr = 4.397(\text{sec})$$

$$Ts = \frac{3.912}{0.5} = 7.824(\text{sec})$$

e) MATLAB produces closed loop system output step responses for K=1



e) MATLAB produces closed loop system output step responses for K=196



comment on the simulated results

For K=1 simulated Ts, Tr and Tp are:

Ts = 6.18 sec; Tp=8.92sec; and Tr = 3.53sec

While for K=196

Ts=0.155sec; Tp=0.25sec; and Tr=0.0883sec

steady-state error for K=1

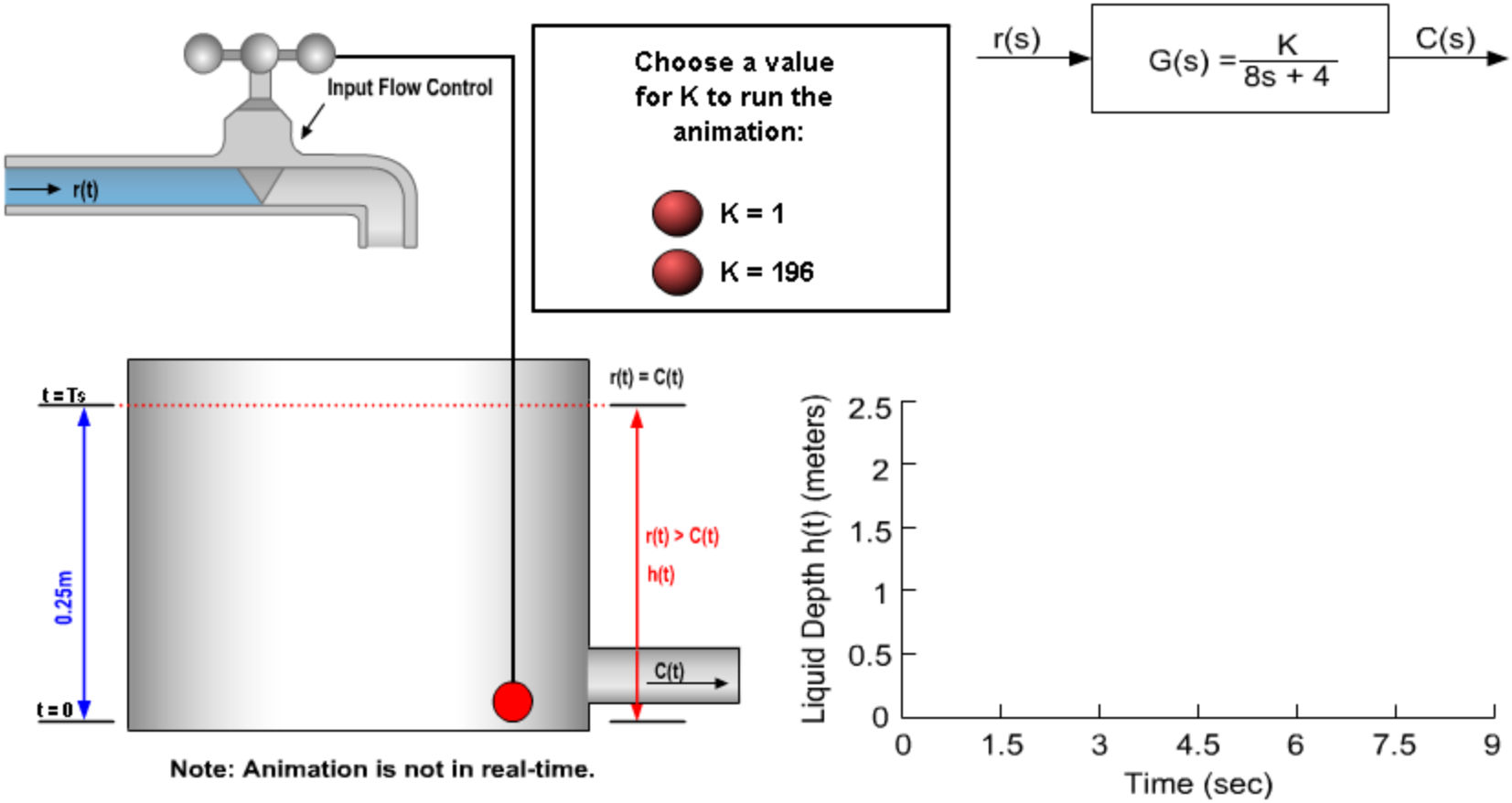
$$e^{\infty}_{STEP} = 80\%$$

and for K=196

$$e^{\infty}_{STEP} = 2\%$$

Animation

This animation demonstrates the system below with the gain values of K = 1 resulting in a steady state error of 80%, and K = 196 which results in a steady state error of 2%.



Note: Animation is not in real-time.

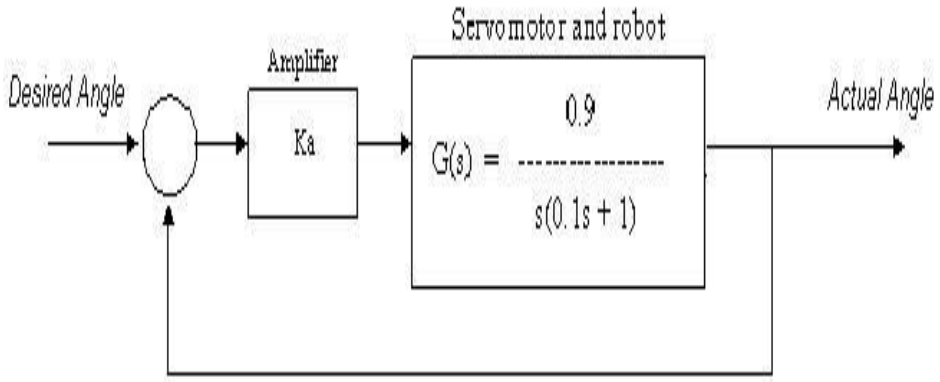
Problem 4

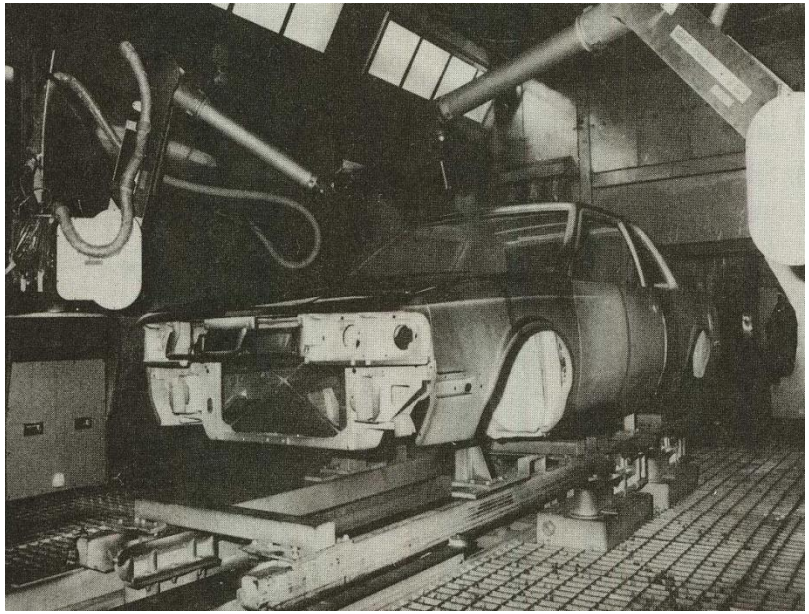
PROBLEM DEFINITION

Manufacturing Robotics

Robotics has revolutionized the manufacturing industry, particularly in the manufacture of automobiles. The image below shows the GM Fanuc Robotics Corporation Model P-150, six-axis articulated arm, electric-servo-driven robot painting an automobile*. The six axes are simultaneously controlled, and driven by state-of-the-art compact ac servomotors. The result is a responsive system with fast acceleration and deceleration, precision painting, and requiring no brush maintenance

Block Diagram





Using MATLAB do the following

1. Find the closed-loop transfer function T(s) for the system described in the figure.
2. Display the step response for unity amplifier gain and step input.
3. Calculate the amplifier gain Ka which will result in a critically damped system

Part 1 & 2

For the unity feedback system given by figure 2 of the question, with

$$G(s) = \frac{0.9}{s(0.1s+1)}$$

and open-loop transfer function

$$G_1(s) = \frac{K_a \times 0.9}{s(0.1s+1)}$$

a

the closed-loop transfer function T(s) is :

$$T(s) = \frac{\frac{K_a \times 0.9}{s(0.1s+1)}}{1 + \frac{K_a \times 0.9}{s(0.1s+1)} \times 1} = \frac{\frac{K_a \times 0.9}{s(0.1s+1)}}{\frac{s(0.1s+1) + K_a \times 0.9}{s(0.1s+1)}} = \frac{K_a \times 0.9}{0.1s^2 + s + 0.9.K_a}$$

or $$T(s) = \frac{9K_a}{s^2 + 10s + 9K_a}$$

Since the roots of the denominator of T(s) are the poles of the closed-loop system, the poles can be expressed as :

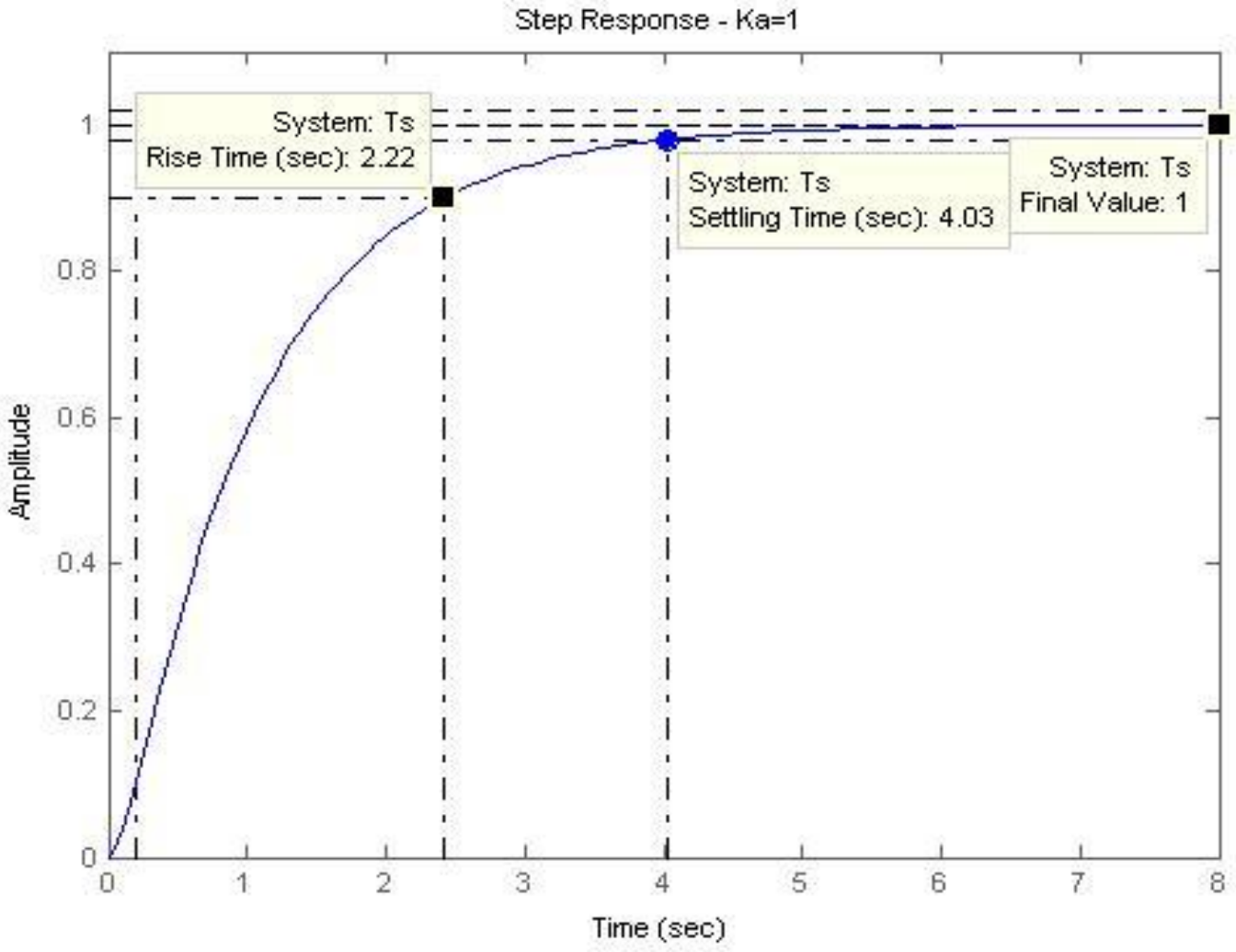
$$Poles = \frac{-10 \pm \sqrt{100 - 36K_a}}{2} = -5 \pm \sqrt{25 - 9K_a}$$

For unity amplifier gain ($K_a = 1$),

$$Poles = -1 \text{ and } -9$$

and $$T(s) = \frac{9}{s^2 + 10s + 9}$$

The step response for system, showing transient response characteristics



Part 3

From part 1 we have the closed-loop poles given by :

$$-5 \pm \sqrt{25 - 9.K_a}$$

where Ka is the amplifier gain.

and the location of the poles for critical damping is :

$$-5$$

Therefore, a system with amplifier gain :

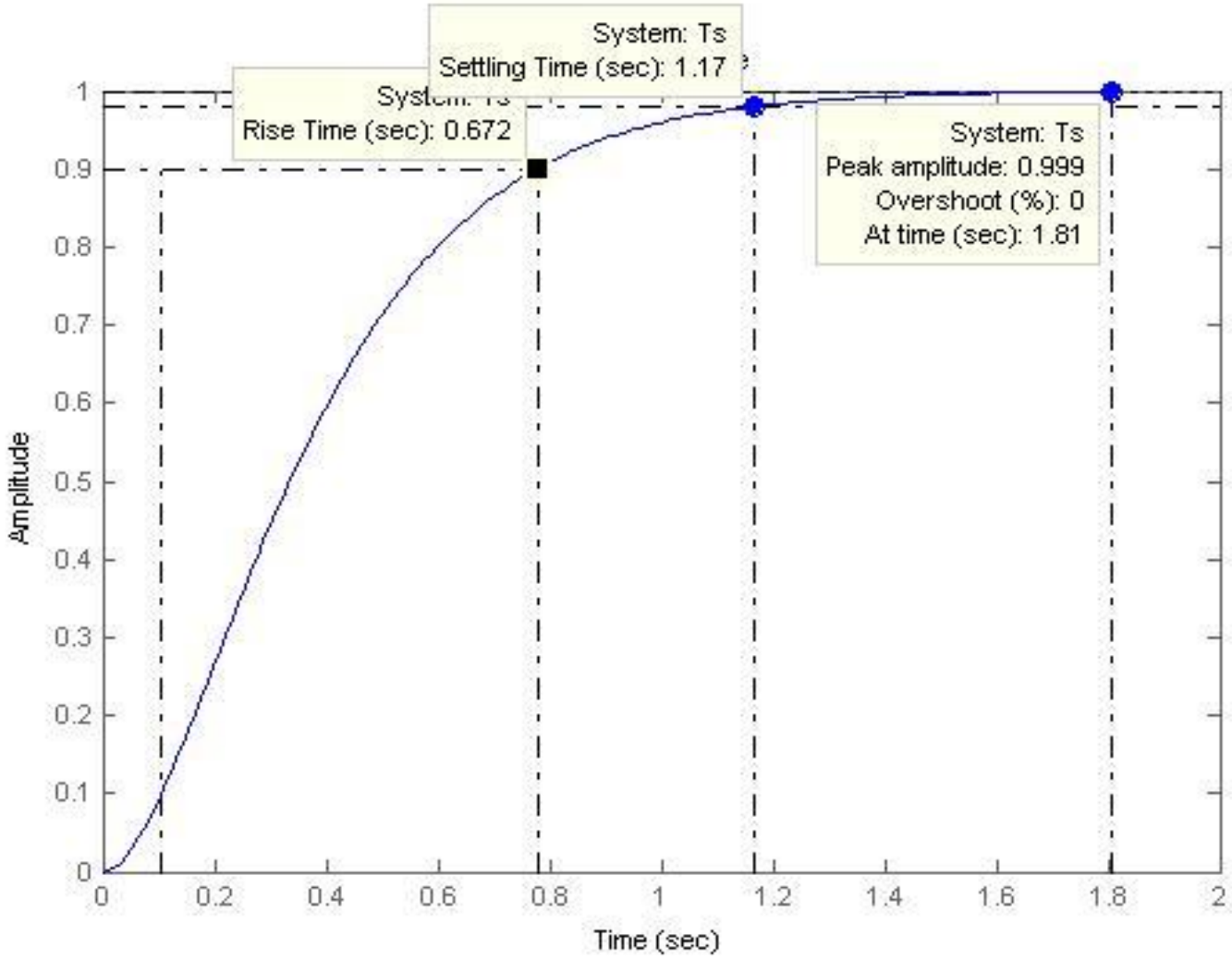
$$0 = \sqrt{25 - 9.K_a}$$

$$25 = 9.K_a$$

$$K_a = 2.\dot{7}7\dot{7}7\dot{8}$$

will be critically damped.

Step response of critically damped system



Animation

Robotic Arm Control System

Select the Animation to Run:

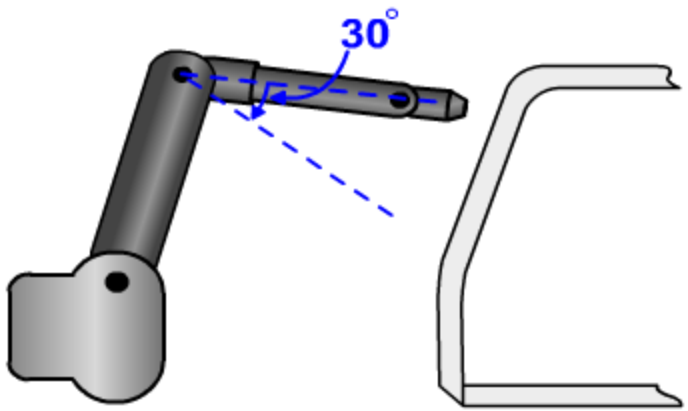
- K=1
- K=2.777
- Control the arm yourself

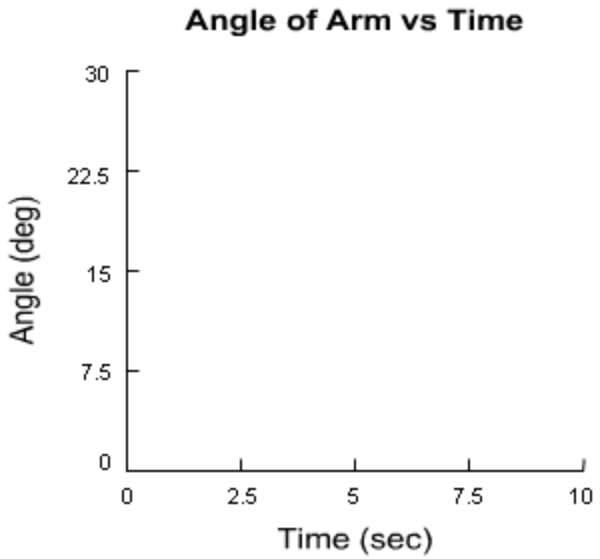
Run | Stop | Reset

Click Run to start Animation

This example represents the control system on one of the robotic arm axes. It will rotate the arm 30 degrees from its current stationary position.

Case 1: A gain of **K = 1**. The result shows an over-damped system, with a settling time of 4 seconds. For the purpose of this application as the problem suggests, a critically damped system is desirable. Therefore a greater gain (K) value is required and this is not the optimal solution.





Note: The animation is not in real time.